November 3, 2014 **Name (Please Print)** ______________________

Optimisation **Final - Semester I 14/15**

1. (20 points) Find an orthonormal basis for the column space of A and using the QR decomposition of A find the least square solution to $Ax = b$ where

$$A = \begin{bmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{bmatrix} \text{ and } b = \begin{bmatrix} -3 \\ 7 \\ 1 \\ 0 \\ 4 \end{bmatrix}.$$

2. (30 points) Decide whether the following statements are true or false (provide adequate justification):

(a) If a basic feasible solution of an LP is degenerate then it corresponds necessarily to two different bases.

(b) A real matrix $A_{n\times n}$ with real eigen values is always orthogonally similar to an upper triangular matrix.

(c) A real $A_{2\times 2}$ matrix is always diagonalisable.

3. (10 points) Consider the affine space

$$W = \left\{ \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix} : \alpha \in \mathbb{R} \right\} \text{ and vector } u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Find the orthogonal projection of u onto W.

4. (20 points) The following is an intermediatory step of the simplex method.

$$\begin{array}{c|cccccc} 1 & 3 & 0 & \frac{-68}{15} & 0 & 0 \\ \hline & & & & & \\ 2 & 2 & 1 & \text{-}3 & 0 & 0 \\ 1 & 1 & 0 & \frac{8}{3} & 1 & 0 \\ 3 & \text{-}1 & 0 & \frac{29}{15} & 0 & 1 \end{array}$$

Write down the current basic feasible solution, its basis, and the cost at this basic feasible solution. Is this solution optimal ? If no, then indicate choice of j and l for the next pivoting step *(do not perform the next pivoting step).*

5. (20 points) Consider the following LP:

$$\begin{aligned} \text{min} \quad & -4x_1 + 2x_2 - 11x_3 + 4x_4 \\ \text{Subject to} \quad & 2x_1 - x_2 + 3x_3 - 2x_4 + x_5 = 6 \\ & x_1 + 4x_2 - x_3 - x_4 + 3x_5 \geq 4 \\ & 4x_1 + 6x_2 - x_3 - 5x_4 + x_5 \geq -8 \\ & x_1 \geq 0, x_2 \in \mathbb{R}, x_3 \geq 0, x_4 \geq 0, x_5 \in \mathbb{R}. \end{aligned}$$

Find the dual of the above LP. Can $x = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \\ -3 \end{bmatrix}$ be a candidate for the optimal solution of the LP ?